# Analyzing Climate Scenarios with Dynamic Mode Decomposition with Control

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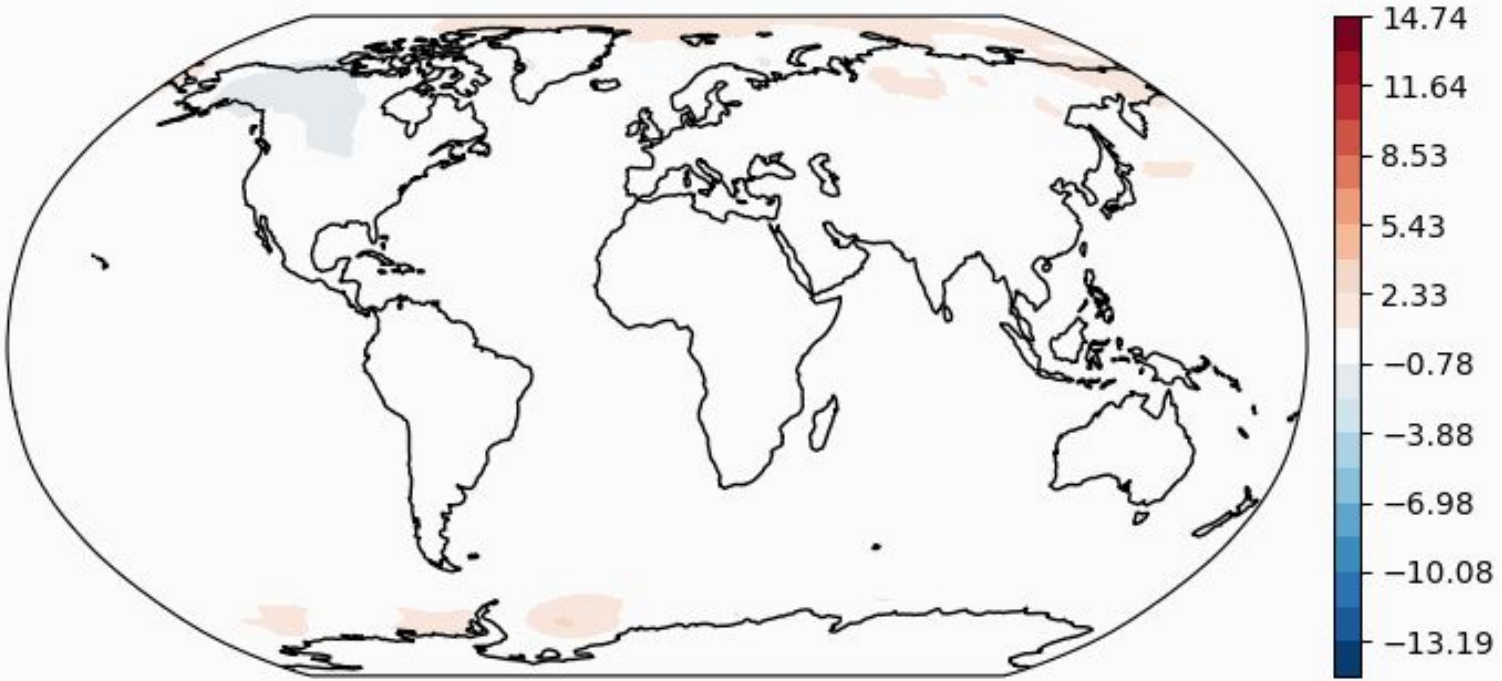
# Outline

- Climate & climate scenarios
- Dimensionality reduction & modes of variability
- Experimental setup & results
- Conclusions & future work

# Climate & climate scenarios

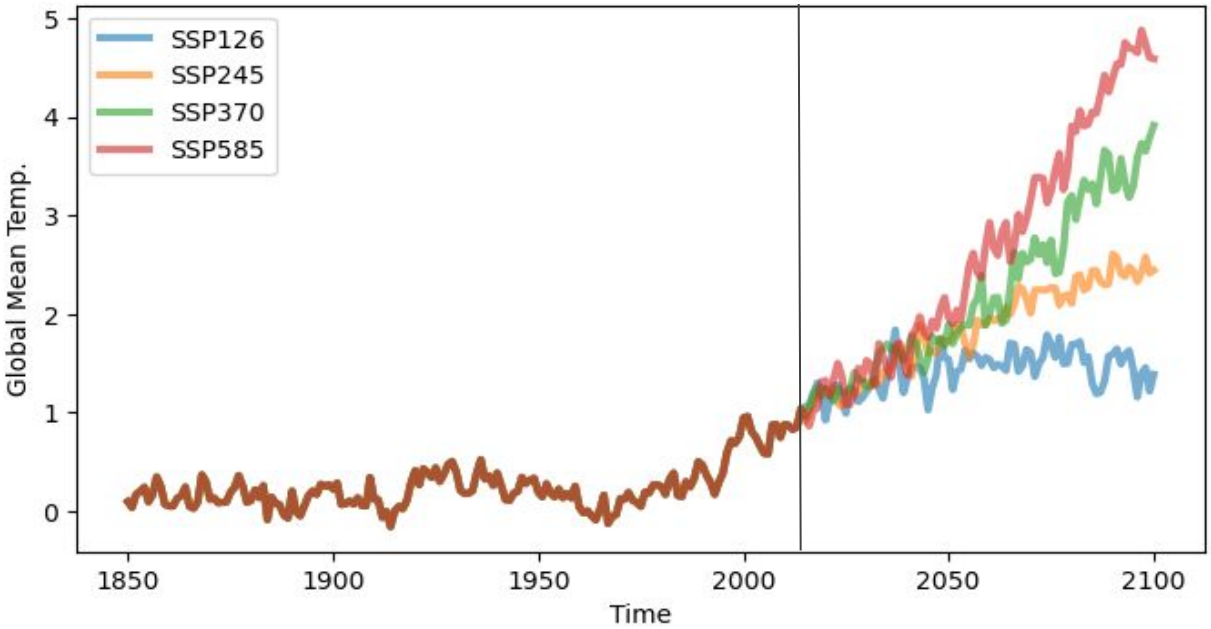
# Climate bench: “high forcing scenario”




Historical temperature (Kelvin) and precipitation data from 1850-2024

(Watson-Parris et. al., 2022)

# Climate bench: mean surface air temperature




Historical data

CMIP6 (Eyring et al. 2016)

SSP data

from ScenarioMIP

# Dimensionality reduction & modes of variability

# Modes of variability

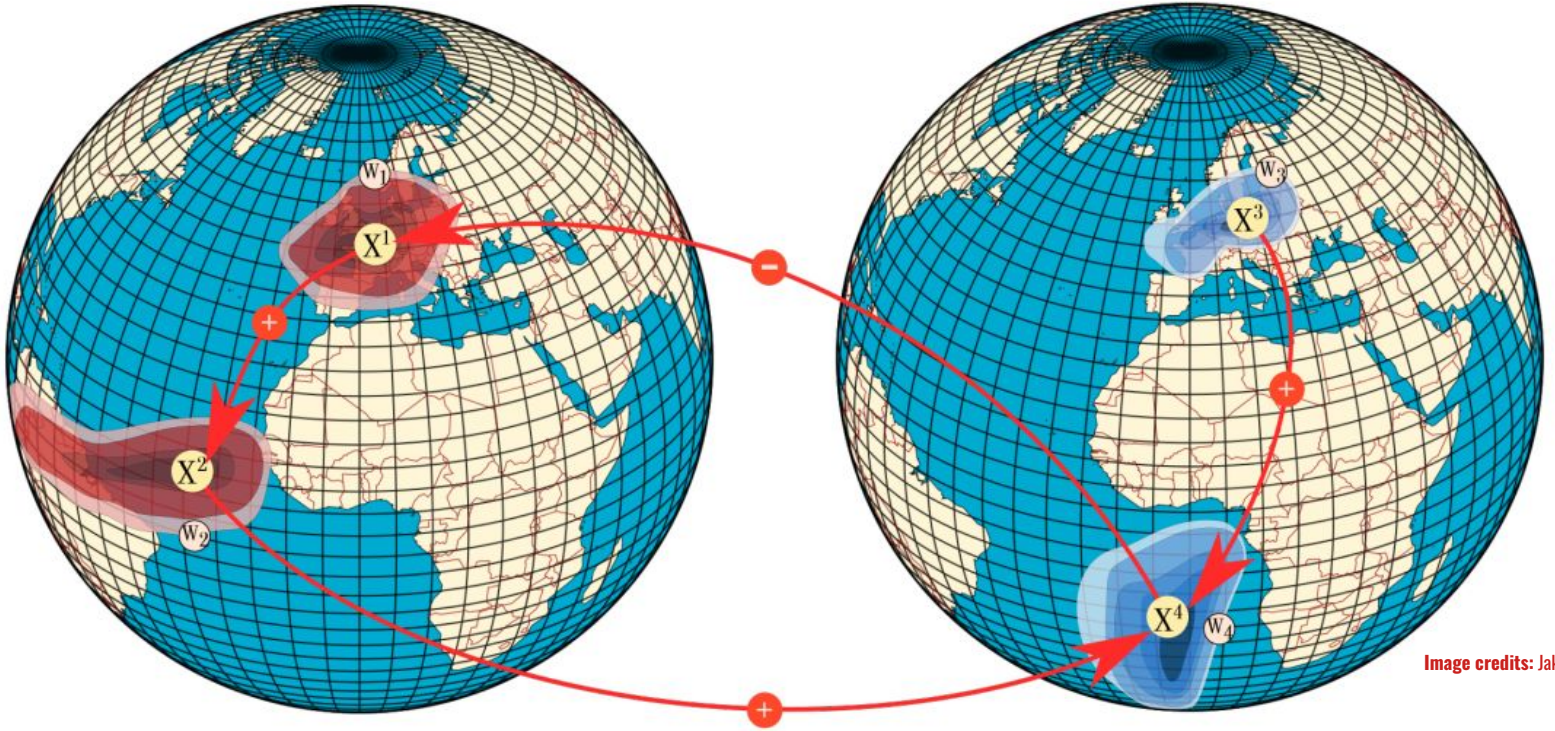



**Image credits:** Jakob Runge, 2019

# Preliminaries

- Time series of flattened global surface air temperatures

$$\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_N\}$$

- Time series of forcing terms (function of (i) carbon dioxide, (ii) methane, (iii) sulfur dioxide, and (iv) black carbon emissions)

$$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_{N-1}\}$$

**Goal:** *Analyze modes of variability across SSPs and compare between methods for finding these modes (e.g., PCA, DMD and DMDc)*

# Methods summary

## PCA (Hotelling, 1936)

**Modes**
Directions of maximum variance

**Eigenvalues**
Explained variance ratio

**Characteristics**
Variance-based analysis

## DMD (Schmidt, 2010)

**Modes**
Spatial contribution

**Eigenvalues**
Real and complex part explains mode evolution over time

**Characteristics**
Linear dynamics-based analysis

## DMDc (Proctor et al., 2016)

**Modes**
See DMD + *forcing modes (from B)*

**Eigenvalues**
See DMD *(no forcing evals)*

**Characteristics**
Same as DMD + linear forcing

# Dynamic Mode Decomposition with control (DMDc)

(Proctor et al., 2016)

*We have a forcing signal- let's use it!*

Data: $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_N\}$ Forcing: $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_{N-1}\}$

Model: $\{\mathbf{A}\mathbf{x}_1+\mathbf{B}\mathbf{u}_1, \mathbf{A}^2(\mathbf{x}_1+\mathbf{B}\mathbf{u}_1)+\mathbf{B}\mathbf{u}_2, \ldots\}$

***Advantages of DMDc***

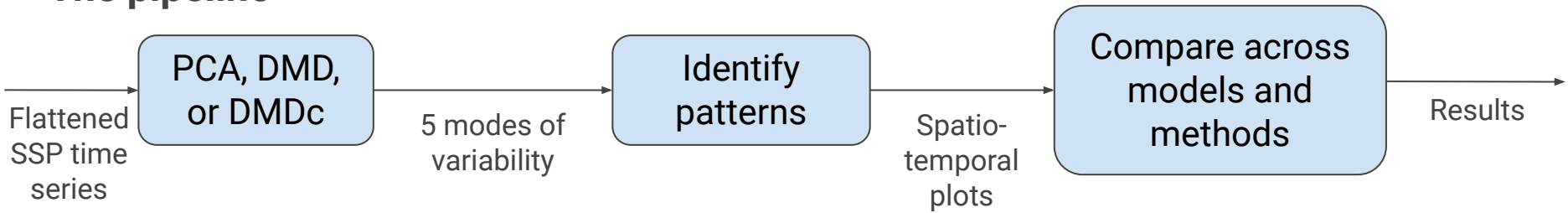
- Forcing added in **Bu**
- Every time step **not** "completely determined by A"
- Spatial modes **without forcing**
- **Forcing spatial modes in B**

Implemented with pyDMD

# Experimental results

# Experimental setup

## The pipeline

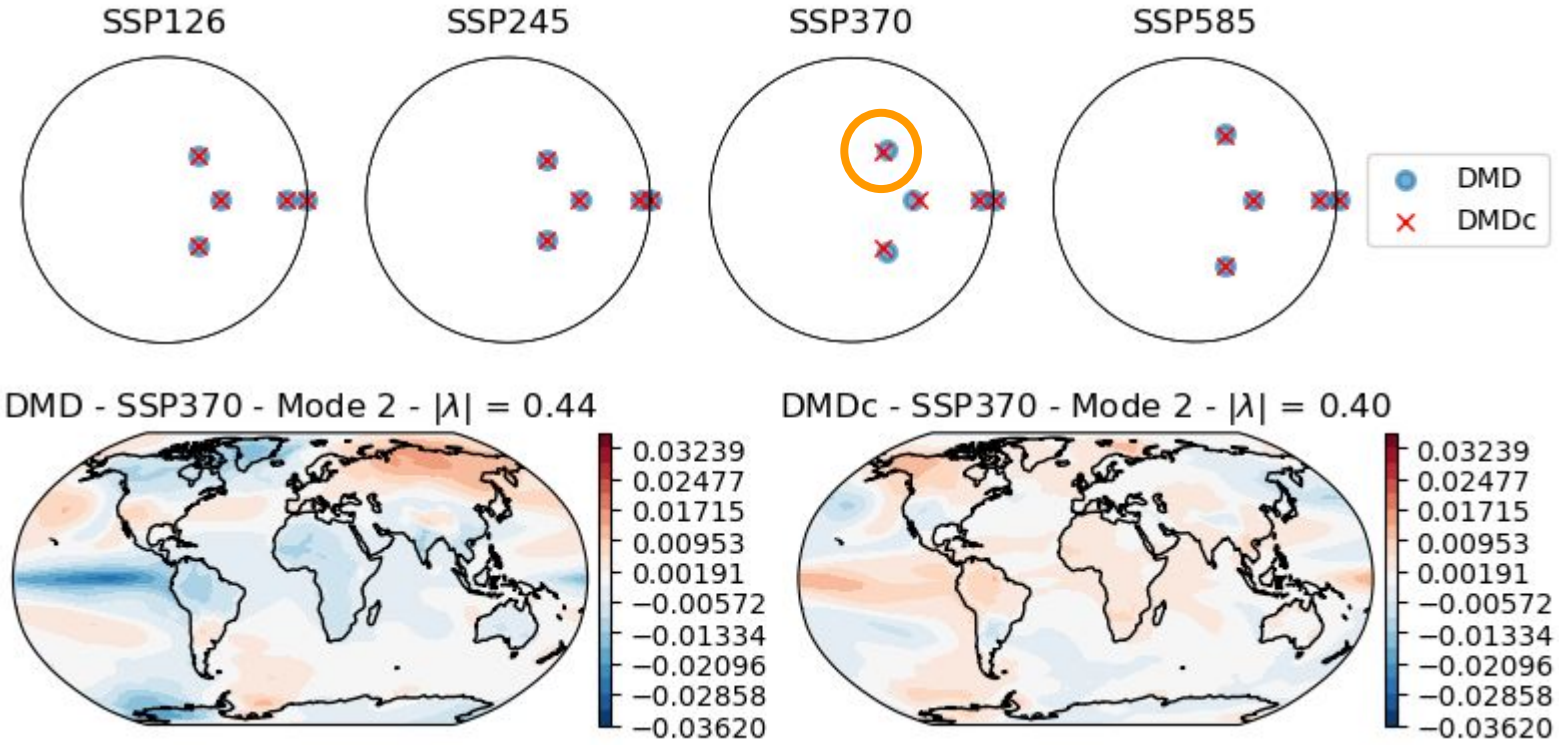



## Our outline

1. Compare DMD to DMDc… they are very similar
2. Identify spatial patterns and their oscillation
3. Modify DMDc parameters to “improve” results

# Experimental results:
## DMD vs DMDc

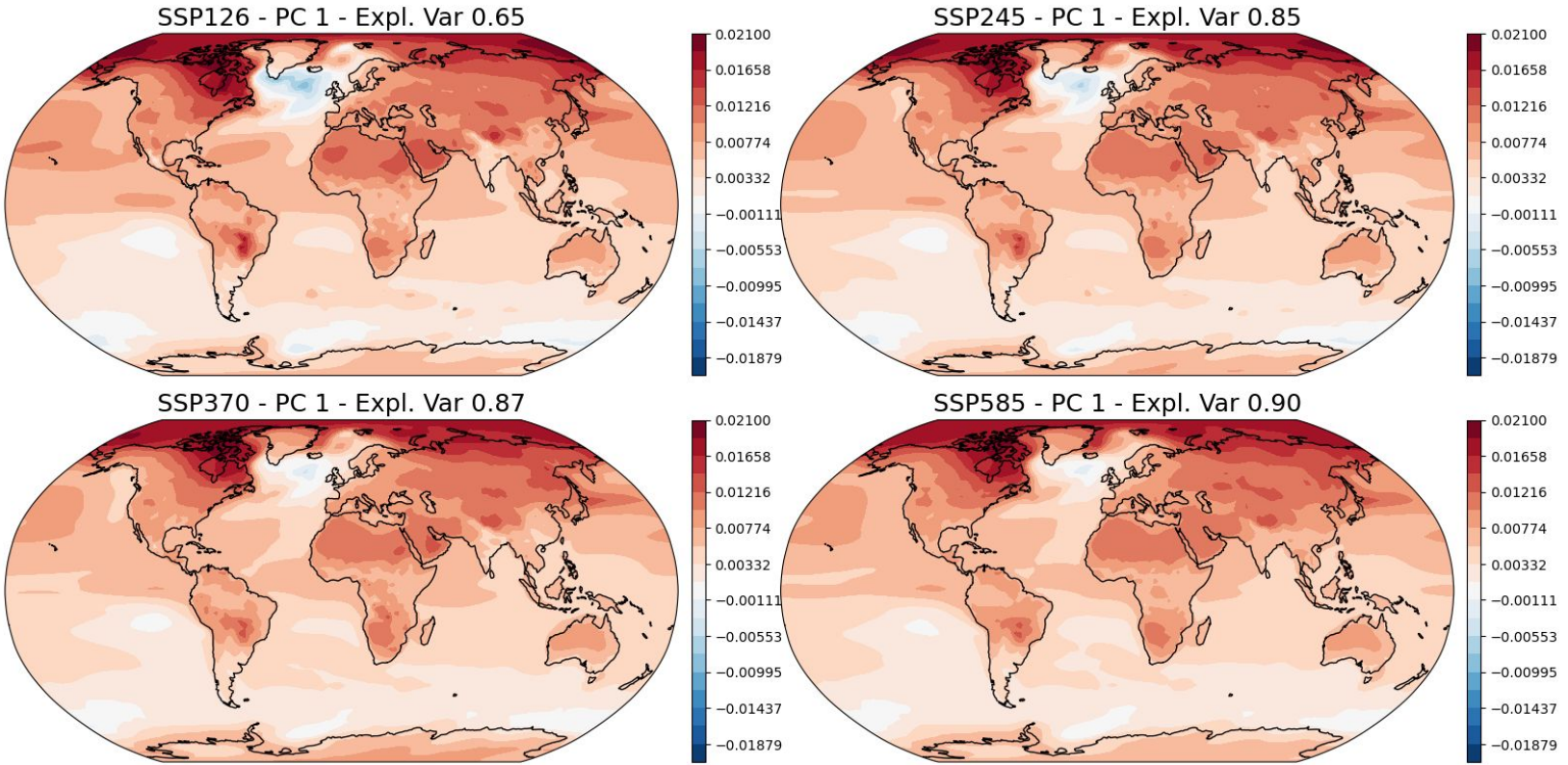
# DMD and DMDc have similar modes



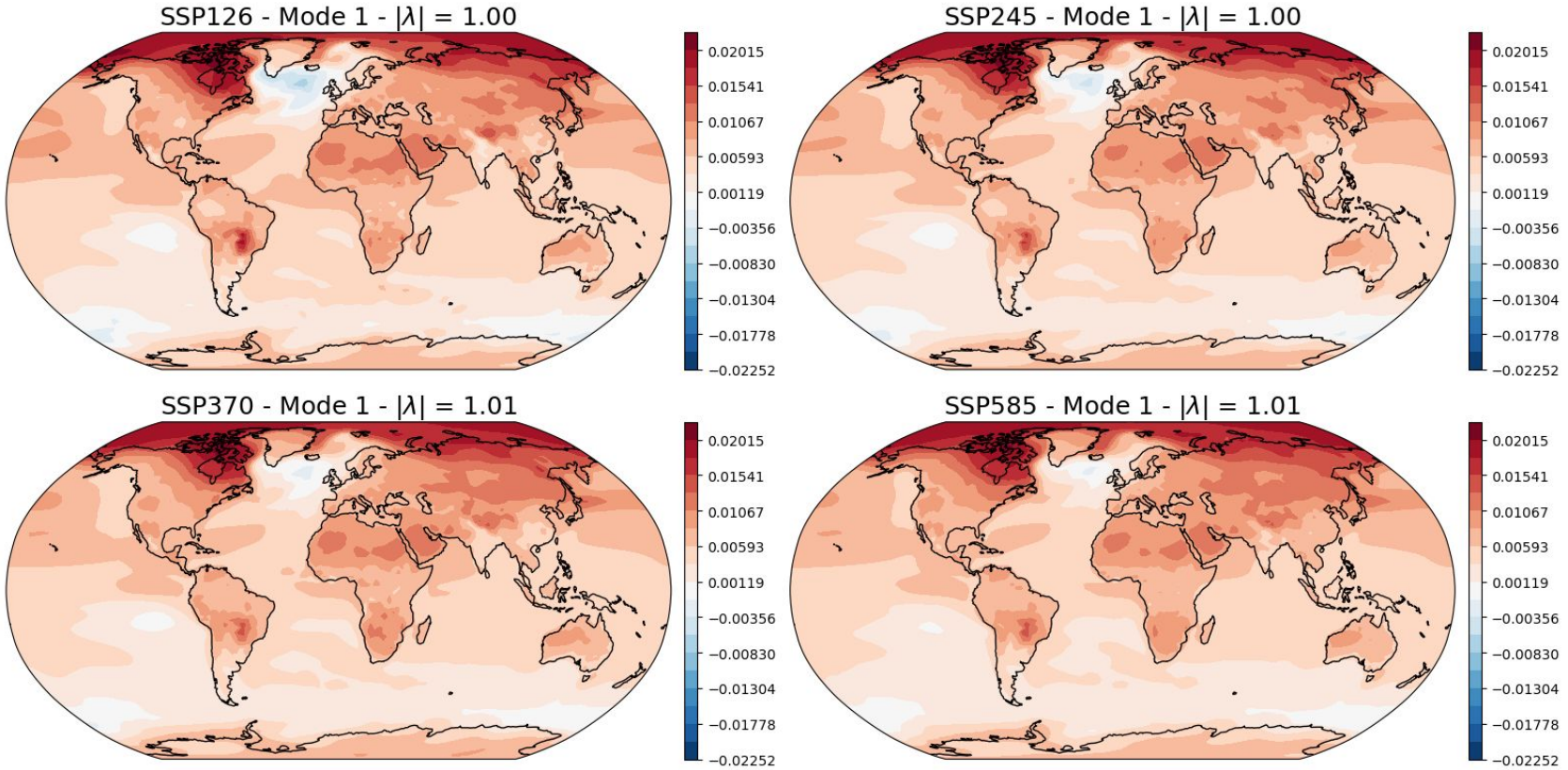

Still similar spatial patterns and almost identical eigenvalues for all SSPs

# Experimental results:
## Modes of variability

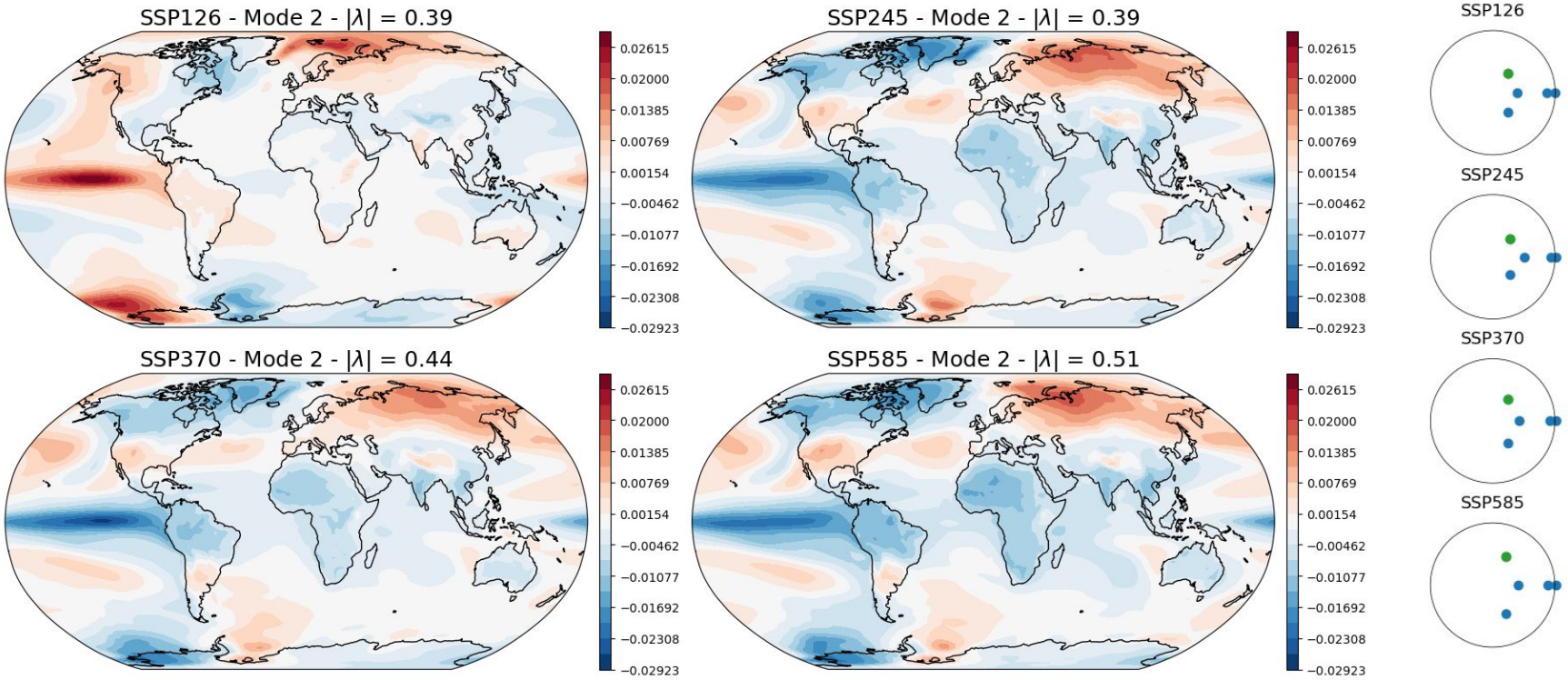
# Warming trend: PCA



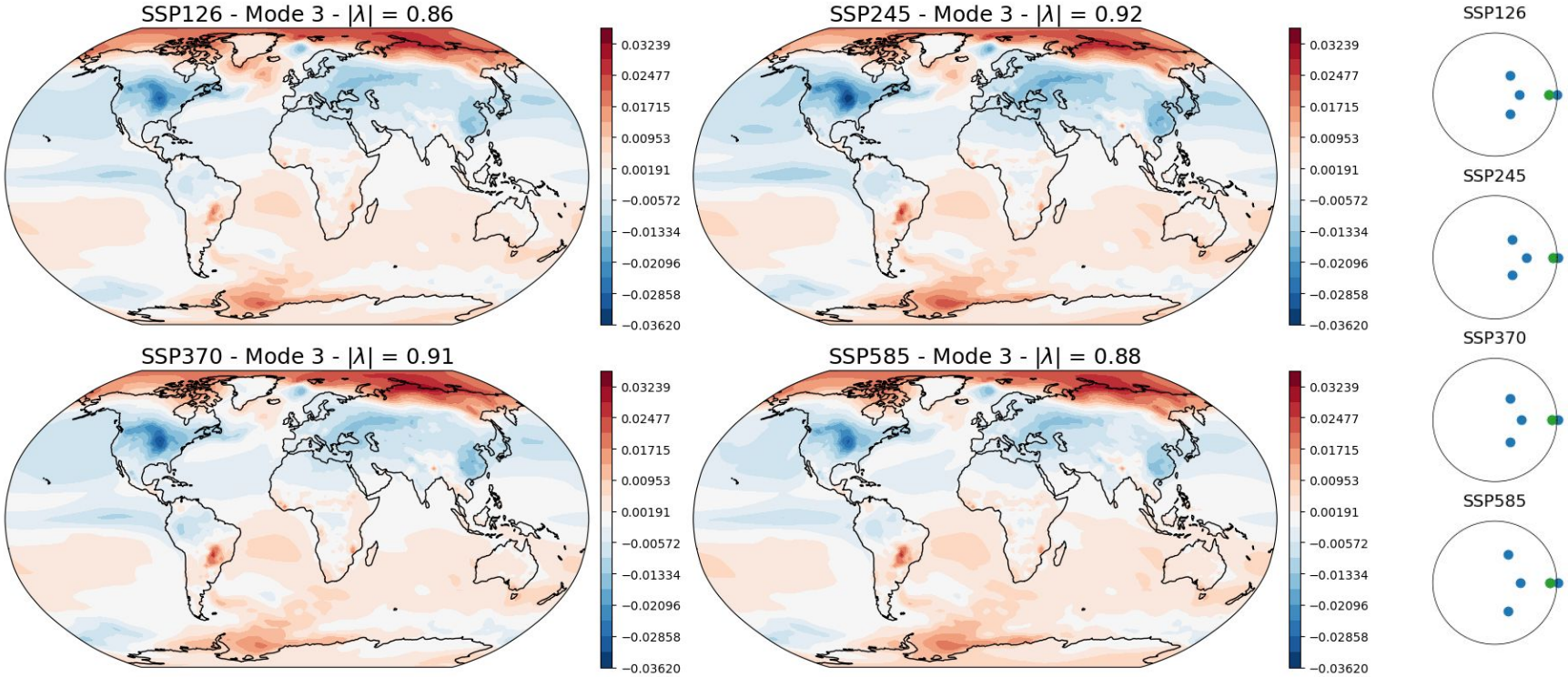
# Warming trend: DMD and DMDc



# El Niño southern oscillation: DMD

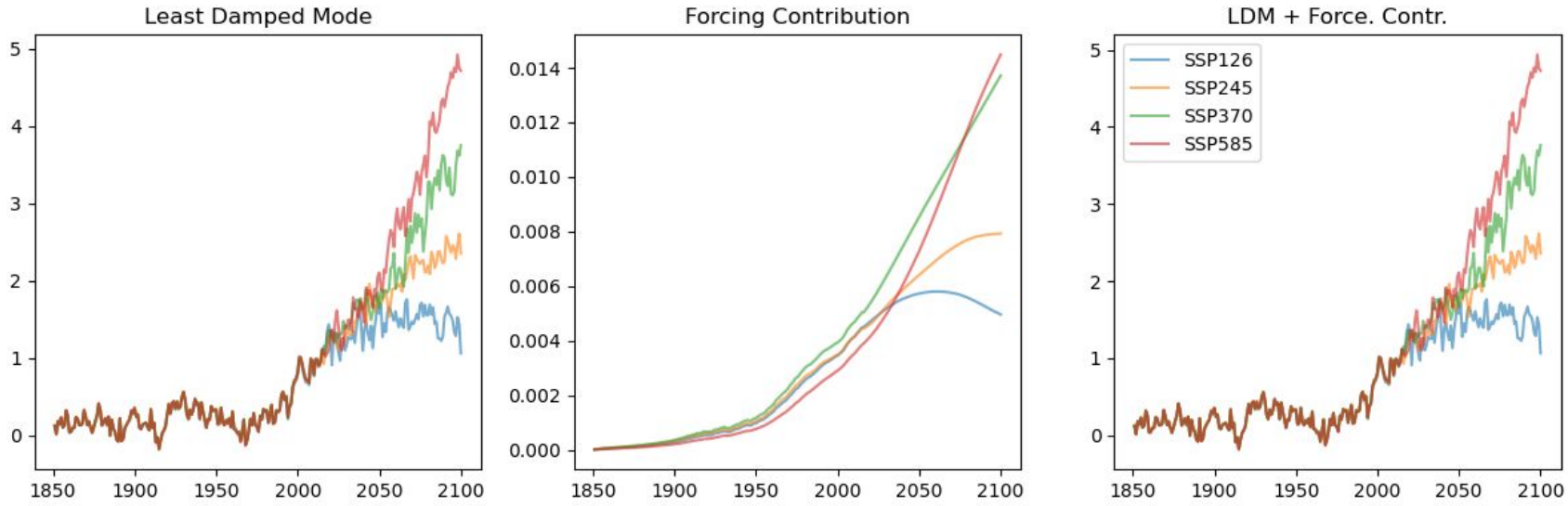


# Contrastive warming & other patterns: DMD



# Experimental results:
## DMDc parameters

# Global mean temperature reconstruction with DMDc

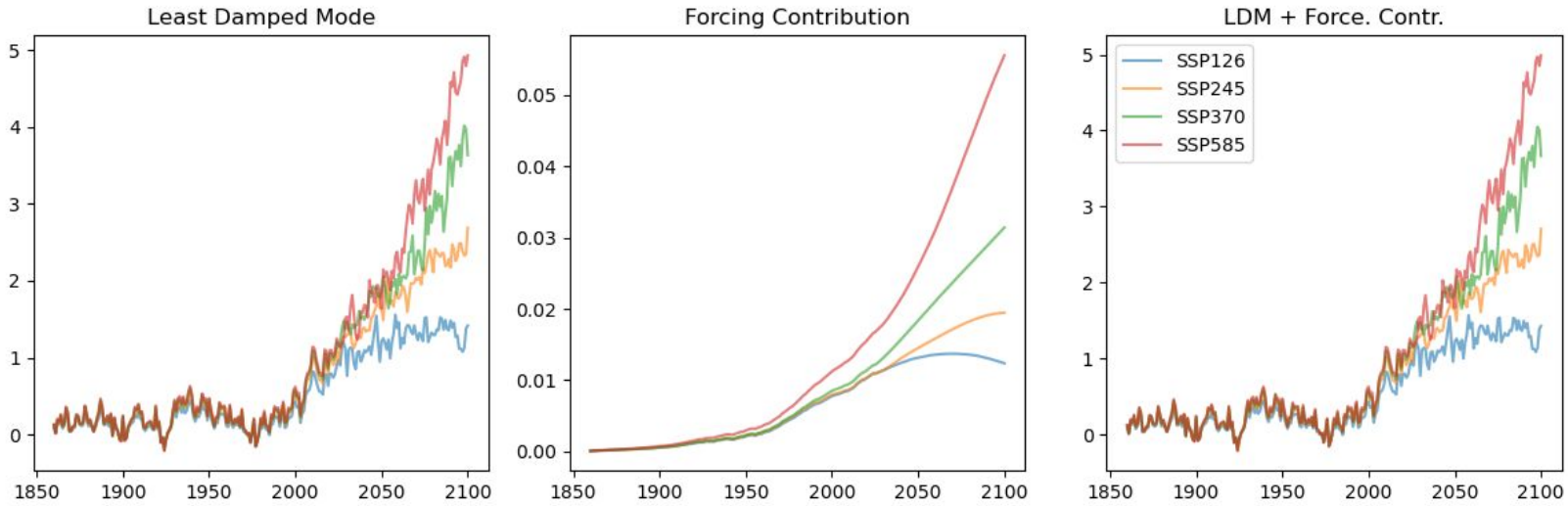



The forcing contribution is **small and similar across SSPs**

# The effect of radiative forcing takes > 1 year

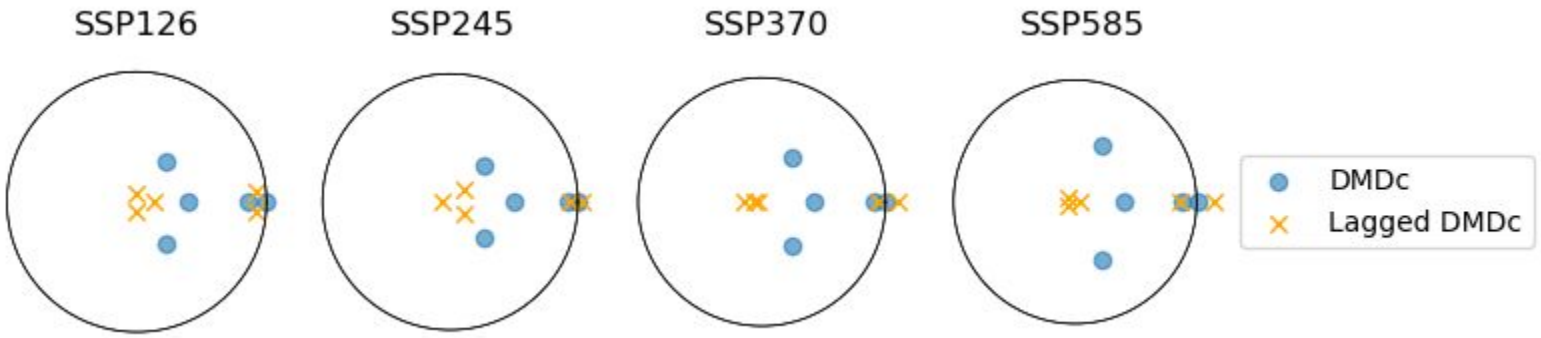
**Idea!** *Forcing effects are lagged by about 10 years*

$$\mathbf{x}_n = \mathbf{A}\mathbf{x}_{n-10} + \mathbf{B}\mathbf{u}_{n-10}$$




**Problem:** less obvious scientific interpretation of spatial modes

# DMDc vs DMDc (time lag 10): Eigenvalues




- Less stable modes in **A** for DMDc
- Spatial patterns of modes are less interpretable
- More dynamics captured in **B**?

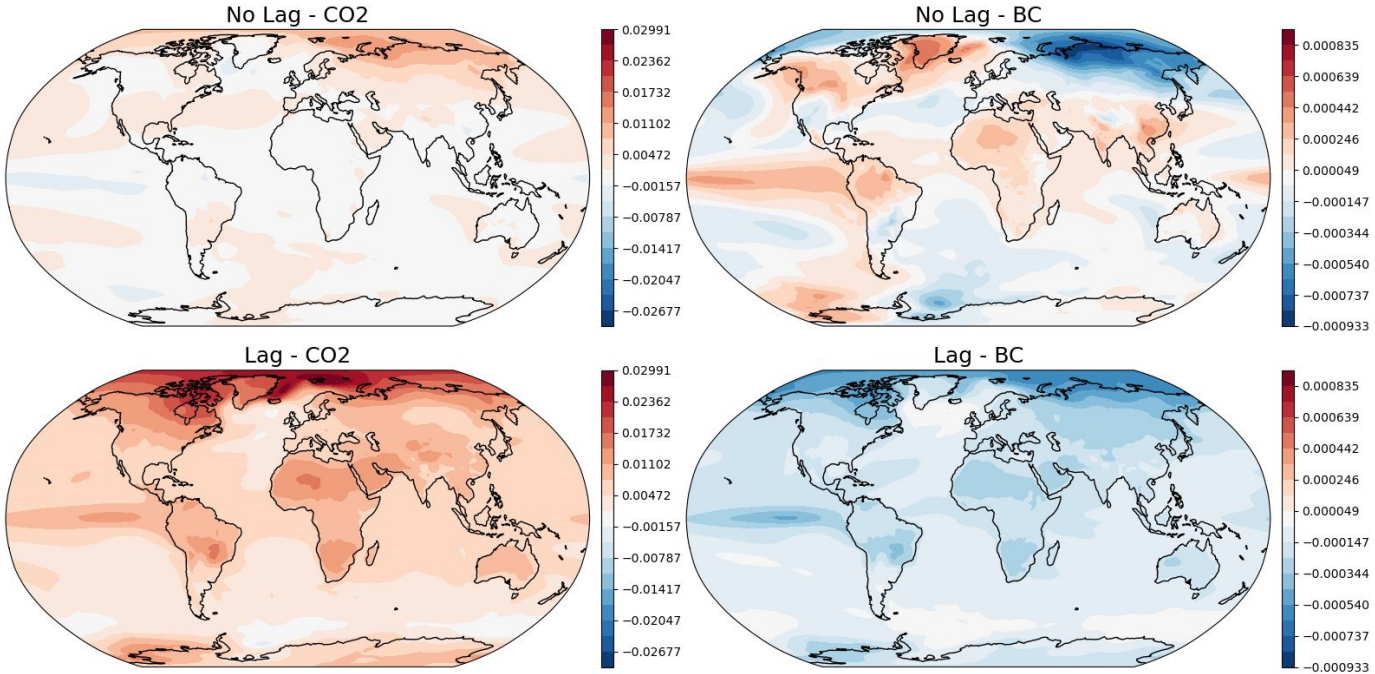
# Forcing contribution from DMDc (SSP 585)

Warming from Carbon

Cooling from Aerosols

Weak signal

Strong signal



# Conclusions & future work

# Concluding remarks

**Take-homes:**

- Re-discovered known modes of variability using PCA, DMD and DMDc
- Time-lagged DMDc improved temperature reconstruction
- Forcing signal from DMDc re-discovered known spatial responses to forcing

**Future Work**

- Collaboration with climate scientists to interpret results
- Causal/ physics aware DMD (Baddoo et al., 2023)
- Optimize DMDc (e.g., time lags)